Anton 8th ed, Ch. 12

12.1

RH system - fingers curve from positive x to positive y axis. Book will use RH system.

Coordinate axis determine 3 coordinate planes: xy-plane, xz-plane, yz-plane

Coordinate planes divide 3-space into 8 parts called octants.

Distance formula, 3-space between $P_1(x_1, y_1, z_1)$ & $P_2(x_2, y_2, z_2)$

$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}$$

Standard eqn of a sphere w/ $C(x_0, y_0, z_0)$ & radius r

$$(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=r^2$$

Thm 12.1.1

An eqn of the form $x^2+y^2+z^2+Gx+Hy+Iz+J=0$
represents a sphere, a point, or has no graph

Graphing eqns in 2-variables in 3-space -
translating a plane curve to some parallel line is called extrusion,
surfaces generated by extrusion are called cylindrical surfaces.

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11a) $(1,0,-1)$, $d=8$
$r=4$

$(x-1)^2+(y-0)^2+(z+1)^2=16$

11c) $(-1,2,1)$ & $(0,2,3)$

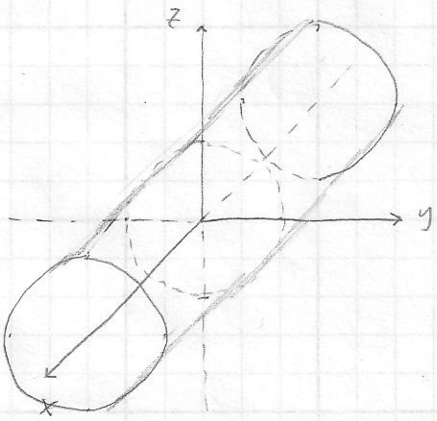
Center = midpt = $(-\frac{1}{2}, 2, 2)$

radius $= \frac{1}{2}d = \frac{1}{2}\sqrt{(-1-0)^2+(2-2)^2+(1-3)^2}$
$= \frac{1}{2}\sqrt{1+0+4} = \frac{1}{2}\sqrt{5}$

$(x+\frac{1}{2})^2+(y-2)^2+(z-2)^2=\frac{5}{4}$

17) $(x^2+10x+5^2)+(y^2+4y+2^2)+(z^2+2z+1^2)=19+25+4+1$
$(x+5)^2+(y+2)^2+(z+1)^2=49$
sphere $c(-5,-2,-1)$
$r=7$

25b) $y^2+z^2=25$



27a) $-2y+z=0$

c) $(x-1)^2+(y-1)^2=1$

12.1

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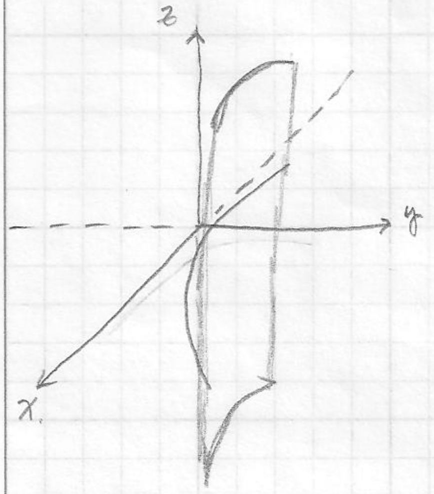
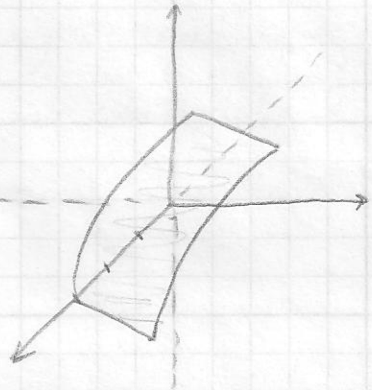
30) $y = e^x$



36) $z = \sqrt{3-x}$



43) $y^2+z^2+6y-4z > 3$

$(y^2+6y+(3)^2]+[z^2-4z+(-2)^2] > 3+9+4$

$(y+3)^2+(z-2)^2 > 16$

all pts outside the circular cylinder given by

$(y+3)^2+(z-2)^2 > 16$

A vector that points in the direction of motion and whose length represents the distance from the starting point to the ending point is called a displacement vector.

Two vectors $\vec{v}$ & $\vec{w}$ are equivalent if they have the same length and the same direction

Components of $\vec{v}$ are written $\vec{v} = \langle v_1, v_2 \rangle$ or $\vec{v} = \langle v_1, v_2, v_3 \rangle$

For two vectors $\vec{v} = \langle v_1, v_2 \rangle$ & $\vec{w} = \langle w_1, w_2 \rangle$

$$\vec{v} \pm \vec{w} = \langle v_1 \pm w_1, v_2 \pm w_2 \rangle \qquad k\vec{v} = \langle kv_1, kv_2 \rangle$$

For two vectors $\vec{v} = \langle v_1, v_2, v_3 \rangle$ & $\vec{w} = \langle w_1, w_2, w_3 \rangle$

$$\vec{v} \pm \vec{w} = \langle v_1 \pm w_1, v_2 \pm w_2, v_3 \pm w_3 \rangle \quad k\vec{v} = \langle kv_1, kv_2, kv_3 \rangle$$

If $\overrightarrow{P_1P_2}$ is a vector in 2-space w/ initial point $P_1(x_1, y_1)$ & terminal point $P_2(x_2, y_2)$ then

$$\overrightarrow{P_1P_2} = \langle x_2 - x_1, y_2 - y_1 \rangle$$

If $\overrightarrow{P_1P_2}$ is a vector in 3-space w/ initial point $P_1(x_1, y_1, z_1)$ & terminal point $P_2(x_2, y_2, z_2)$ then

$$\overrightarrow{P_1P_2} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$$

For any vectors $\vec{u}, \vec{v}, \& \vec{w}$ and any scalars l & k

a) $\vec{u} + \vec{v} = \vec{v} + \vec{u}$
b) $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$
c) $\vec{u} + \vec{0} = \vec{0} + \vec{u} = \vec{u}$
d) $\vec{u} + (-\vec{u}) = \vec{0}$
e) $k(l\vec{u}) = (kl)\vec{u}$
f) $k(\vec{u} + \vec{v}) = k\vec{u} + k\vec{v}$
g) $(k + l)\vec{u} = k\vec{u} + l\vec{u}$
h) $1\vec{u} = \vec{u}$

The distance between the initial & terminal point of a vector $\vec{v}$ is called the length, norm, or magnitude of $\vec{v}$ and is denoted by $\|\vec{v}\|$

$$\|k\vec{v}\| = |k|\,\|\vec{v}\|$$

A unit vector is a vector of length 1

in 2-space $\vec{i} = \langle 1, 0 \rangle$, $\vec{j} = \langle 0, 1 \rangle$ $\Rightarrow \vec{v} = \langle v_1, v_2 \rangle = v_1\vec{i} + v_2\vec{j}$

in 3-space $\vec{i} = \langle 1, 0, 0 \rangle$, $\vec{j} = \langle 0, 1, 0 \rangle$, $\vec{k} = \langle 0, 0, 1 \rangle$
$\Rightarrow \vec{v} = \langle v_1, v_2, v_3 \rangle = v_1\vec{i} + v_2\vec{j} + v_3\vec{k}$

The process of multiplying a vector by the reciprocal of its length is called normalizing $\vec{v}$

$$\vec{u} = \frac{\vec{v}}{\|\vec{v}\|}$$

Vector determined by length & angle

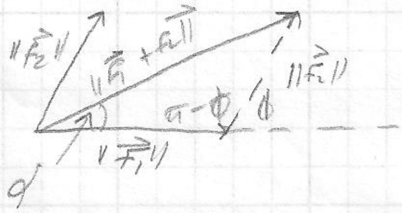
$\vec{v} = \|\vec{v}\| \langle \cos\phi, \sin\phi \rangle$ ϕ is the angle from the positive x-axis to the radial line of $\vec{v}$

$= \|\vec{v}\| \cos\phi \, \vec{i} + \|\vec{v}\| \sin\phi \, \vec{j}$

Two forces $\vec{F_1}$ & $\vec{F_2}$ applied at the same point on an object have the same effect on the object as the single force $\vec{F_1} + \vec{F_2}$ (called the <u>resultant</u> of $\vec{F_1}$ & $\vec{F_2}$.

The forces $\vec{F_1}$ & $\vec{F_2}$ are <u>concurrent</u> if they are applied at the same point.

$\|\vec{F_1} + \vec{F_2}\|^2 = \|\vec{F_1}\|^2 + \|\vec{F_2}\|^2 + 2\|F_1\|\|F_2\|\cos\phi$



$\sin\theta = \dfrac{\|\vec{F_2}\|}{\|\vec{F_1} + \vec{F_2}\|} \sin\phi$

<u>P. 801</u>

8a) $\vec{P_1P_2} = \langle -4+6, -1+2 \rangle = \underline{\langle 2, 1 \rangle}$

c) $\vec{P_1P_2} = \langle 9-4, 1-1, -3+3 \rangle = \underline{\langle 5, 0, 0 \rangle}$

14d) $\vec{v} = \vec{i} + \vec{j} + \vec{k}$

$\|\vec{v}\| = \sqrt{1^2 + 1^2 + 1^2}$

$= \underline{\sqrt{3}}$

18b) $2\vec{i} - \vec{j} - 2\vec{k}$

$\|2\vec{i} - \vec{j} - 2\vec{k}\| = \sqrt{4+1+4}$

$= 3$

$\therefore \frac{1}{3}(2\vec{i} - \vec{j} - 2\vec{k})$

$= \frac{2}{3}\vec{i} - \frac{1}{3}\vec{j} - \frac{2}{3}\vec{k}$

23) $\vec{v} = \|\vec{v}\| \langle \cos\phi, \sin\phi \rangle$

$\dfrac{\vec{v}}{\|\vec{v}\|} = \langle \cos\phi, \sin\phi \rangle$

$\dfrac{\vec{v}}{\|\vec{v}\|} = \langle \cos\frac{\pi}{6}, \sin\frac{\pi}{6} \rangle = \langle \frac{\sqrt{3}}{2}, \frac{1}{2} \rangle$

$\vec{w} = \|\vec{w}\| \langle \cos\phi, \sin\phi \rangle$

$\dfrac{\vec{w}}{\|\vec{w}\|} = \langle \cos\phi, \sin\phi \rangle$

$\dfrac{\vec{w}}{\|\vec{w}\|} = \langle \cos\frac{3\pi}{4}, \sin\frac{3\pi}{4} \rangle = \langle -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \rangle$

$\therefore \vec{v} + \vec{w} = \underline{\left\langle \frac{\sqrt{3} - \sqrt{2}}{2}, \frac{1 + \sqrt{2}}{2} \right\rangle}$

31)

$\vec{v} = \vec{i} + \vec{j}$ $\vec{w} = \vec{i} - 2\vec{j}$

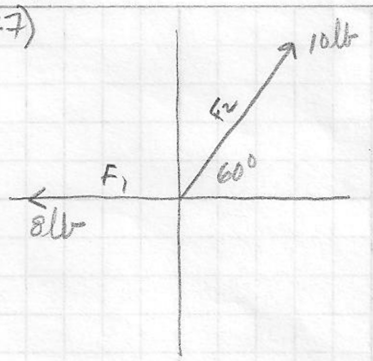
$\vec{v} + \vec{w} = 2\vec{i} - \vec{j}$

$\|\vec{v} + \vec{w}\| = \sqrt{4+1} = \underline{\sqrt{5}}$

$\|\vec{v} - \vec{w}\| = \sqrt{4-1} = \underline{\sqrt{3}}$

Ch.12 12.2

47)



$$\|\vec{F}_1 + \vec{F}_2\| = \sqrt{8^2 + 10^2 + 2(8)(10)\cos 120}$$
$$= \sqrt{84}$$
$$= 2\sqrt{21}$$
$$\approx \underline{9.165\text{ lb}}$$

$$60^\circ + \sin^{-1} \frac{\|\vec{F}_1\|}{\|\vec{F}_1 + \vec{F}_2\|} \sin 120^\circ$$

$\approx 109^\circ$

∴ To achieve static equilibrium, have the resultant pull 180° opposite

$\vec{F} \approx 9.165$ lb, $\alpha \approx -70^\circ$

12.3

Dot Product:

$\vec{u} = \langle u_1, u_2 \rangle$ & $\vec{v} = \langle v_1, v_2 \rangle$ $\qquad$ $\vec{u} = \langle u_1, u_2, u_3 \rangle$, $\vec{v} = \langle v_1, v_2, v_3 \rangle$

$\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2$ $\qquad$ $\vec{u} \cdot \vec{v} = u_1 v_1 + u_2 v_2 + u_3 v_3$

Thm 12.3.2

$\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$

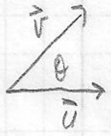
$\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$

$k(\vec{u} \cdot \vec{v}) = (k\vec{u}) \cdot \vec{v} = \vec{u} \cdot (k\vec{v})$

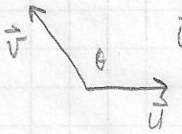
$\vec{v} \cdot \vec{v} = \|\vec{v}\|^2$ $\qquad \Rightarrow \|\vec{v}\| = \sqrt{\vec{v} \cdot \vec{v}}$

$\vec{0} \cdot \vec{v} = 0$

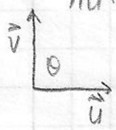
The angle between vector $\vec{u}$ & $\vec{v}$ $\quad \cos\theta = \dfrac{\vec{u} \cdot \vec{v}}{\|\vec{u}\|\|\vec{v}\|}$ or $\vec{u} \cdot \vec{v} = \|\vec{u}\|\|\vec{v}\| \cos\theta$



$\vec{u} \cdot \vec{v} > 0$



$\vec{u} \cdot \vec{v} < 0$



$\vec{u} \cdot \vec{v} = 0$

The angles between a nonzero vector $\vec{v}$ and the vector $\vec{i}, \vec{j}, \& \vec{k}$ are called the direction angles of $\vec{v}$, the cosines of those angles are called the direction cosines of $\vec{v}$.

The direction cosines for nonzero vector $\vec{v} = v_1 \vec{i} + v_2 \vec{j} + v_3 \vec{k}$ are

$\cos\alpha = \dfrac{v_1}{\|\vec{v}\|}$, $\cos\beta = \dfrac{v_2}{\|\vec{v}\|}$, $\cos\gamma = \dfrac{v_3}{\|\vec{v}\|}$

The direction cosines satisfy $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

The downward force $\vec{F}$ that gravity exerts on an object can be decomposed into the sum $\vec{F} = \vec{F}_1 + \vec{F}_2$

F_1 is parallel to a ramp, it is the force that pulls the object along the ramp

F_2 is perpendicular to the ramp, F_2 is the force that the block exerts against the ramp.

the orthogonal projection of $\vec{v}$ on $\vec{b}$

$proj_{\vec{b}} \vec{v} = \dfrac{\vec{v} \cdot \vec{b}}{\|b\|^2} \vec{b}$

$\vec{v} - proj_{\vec{b}} \vec{v}$ the vector component of $\vec{v}$ orthogonal to $\vec{b}$

12.3

Work

$W = F \times d$ = force × distance

$\|\vec{F}\| = F \qquad \|\vec{PQ}\| = d$

$W = \|\vec{F}\|\,\|\vec{PQ}\|$

If $\vec{F}$ is not in the direction of motion, but rather makes an angle θ with the displacement vector

$$W = (\|\vec{F}\| \cos\theta)\,\|\vec{PQ}\| = \vec{F}\cdot\vec{PQ}$$

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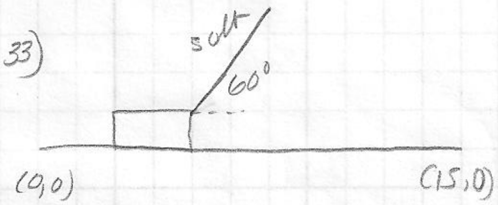
3b) $\vec{u} = 6\vec{i} + \vec{j} + 3\vec{k}$, $\vec{v} = 4\vec{i} - 6\vec{k}$

$\vec{u}\cdot\vec{v} = (6)(4) + (1)(0) + (3)(-6)$
$= 24 + 0 - 18$
$= 6$ acute

33)



$\vec{PQ} = 15\vec{i}$

$\vec{F} = 50\cos 60\,\vec{i} + 50\sin 60\,\vec{j}$
$\vec{F} = 25\vec{i} + \frac{25}{\sqrt{3}}\vec{j}$

$W = \vec{F}\cdot\vec{PQ} = \left(25\vec{i} + \frac{25}{\sqrt{3}}\vec{j}\right)\cdot\left(15\vec{i}\right)$
$= 375$ ft-lb

15a) $\vec{v} = \vec{i} + \vec{j} - \vec{k}$

$\cos\alpha = \frac{1}{\sqrt{3}} \approx 55°$

$\cos\beta = \frac{1}{\sqrt{3}} \approx 55°$

$\cos\gamma = \frac{-1}{\sqrt{3}} \approx 125°$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{-1}{\sqrt{3}}\right)^2 = 1$

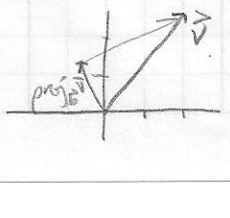
$\frac{1 + 1 + 1}{3} = 1$

$\frac{3}{3} = 1$ ✓

21b) $\vec{v} = \langle 4,5\rangle$, $\vec{b} = \langle 1,-2\rangle$

$\text{proj}_{\vec{b}}\vec{v} = \frac{(4-10)}{(\sqrt{5})^2}\langle 1,-2\rangle = \left\langle -\frac{6}{5}, \frac{12}{5}\right\rangle$

$\vec{v} - \text{proj}_{\vec{b}}\vec{v} = \left\langle \frac{26}{5}, \frac{13}{5}\right\rangle$



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12.4

Determinant - fns that assign numerical values to square arrays of numbers

Note:

(a) If 2 rows in the array are the same, then the det = 0

(b) Interchanging 2 rows in the array of a det multiplies its value by -1

Cross Product - If $\vec{u} = \langle u_1, u_2, u_3 \rangle$ & $\vec{v} = \langle v_1, v_2, v_3 \rangle$ are vectors in 3-space, then the cross product $\vec{u} \times \vec{v}$ is defined by

$$\vec{u} \times \vec{v} = \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} \vec{i} - \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix} \vec{j} + \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix} \vec{k} \quad * \text{ or equivalently}$$

$$\vec{u} \times \vec{v} = (u_2 v_3 - u_3 v_2)\vec{i} - (u_1 v_3 - u_3 v_1)\vec{j} + (u_1 v_2 - u_2 v_1)\vec{k}$$

$$* \quad \vec{u} \times \vec{v} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$$

Note: The cross product is defined only for vectors in 3-space; dot product is only defined in 2-space

The cross product of 2 vectors is a vector, the dot product of two vectors is a scalar.

Algebraic properties - If $\vec{u}$, $\vec{v}$, & $\vec{w}$ are any vectors in 3 space & k is any scalar, then

(a) $\vec{u} \times \vec{v} = -(\vec{v} \times \vec{u})$

(b) $\vec{u} \times (\vec{v} + \vec{w}) = \vec{u} \times \vec{v} + \vec{u} \times \vec{w}$

(c) $(\vec{u} + \vec{v}) \times \vec{w} = (\vec{u} \times \vec{w}) + (\vec{v} \times \vec{w})$

(d) $k(\vec{u} \times \vec{v}) = (k\vec{u}) \times \vec{v} = \vec{u} \times (k\vec{v})$

(e) $\vec{u} \times \vec{0} = \vec{0} \times \vec{u} = \vec{0}$

(f) $\vec{u} \times \vec{u} = \vec{0}$

Note: $\vec{i} \times \vec{j} = \vec{k}$ $\quad \vec{j} \times \vec{k} = \vec{i}$ $\quad \vec{k} \times \vec{i} = \vec{j}$

$\vec{j} \times \vec{i} = -\vec{k}$ $\quad \vec{k} \times \vec{j} = -\vec{i}$ $\quad \vec{i} \times \vec{k} = -\vec{j}$

If $\vec{u}$ & $\vec{v}$ are vectors in 3-space, then

(a) $\vec{u} \cdot (\vec{u} \times \vec{v}) = 0$ $\quad$ ($\vec{u} \times \vec{v}$ is orth to $\vec{u}$)

(b) $\vec{v} \cdot (\vec{u} \times \vec{v}) = 0$ $\quad$ ($\vec{u} \times \vec{v}$ is orth to $\vec{v}$)

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12.4

Let $\vec{u}$ & $\vec{v}$ be nonzero vectors in 3-space, and let θ be the angle between these vectors when they are positioned so their initial point coincide.

(a) $\|\vec{u} \times \vec{v}\| = \|\vec{u}\|\|\vec{v}\| \sin\theta$

(b) The Area of a parallelogram that has $\vec{u}$ & $\vec{v}$ as adjacent sides is

$$A = \|\vec{u} \times \vec{v}\|$$

(c) $\vec{u} \times \vec{v} = \vec{0}$ iff $\vec{u}$ & $\vec{v}$ are parallel vectors, that is, iff they are scalar multiple of one another

Triple scalar product $\quad \vec{u} \cdot (\vec{v} \times \vec{w}) = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$

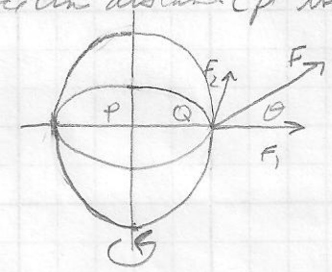
Let $\vec{u}$, $\vec{v}$ and $\vec{w}$ be nonzero vectors in 3-space.

(a) The Volume of the parallelpiped that has $\vec{u}, \vec{v}, \& \vec{w}$ as adjacent edges is

$$V = |\vec{u} \cdot (\vec{v} \times \vec{w})|$$

(b) $\vec{u} \cdot (\vec{v} \times \vec{w}) = 0$ iff $\vec{u}, \vec{v}, \& \vec{w}$ lie in the same plane.

Scalar moment or torque of $\vec{F}$ $\quad \|\overrightarrow{PQ}\|\|\vec{F}\| \sin\theta = \|\overrightarrow{PQ} \times \vec{F}\|$; they have units of force times distance [ft-lb, N·m, etc.]



The vector $\overrightarrow{PQ} \times \vec{F}$ is called the vector moment of torque vector of $\vec{F}$ @ P

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6) $\vec{u} = 4\vec{i} + \vec{k}$, $\vec{v} = 2\vec{i} - \vec{j}$

$\langle 4, 0, 1 \rangle \qquad \langle 2, -1, 0 \rangle$

$\vec{u} \times \vec{v} = \begin{vmatrix} 0 & 1 \\ -1 & 0 \end{vmatrix} \vec{i} - \begin{vmatrix} 4 & 1 \\ 2 & 0 \end{vmatrix} \vec{j} + \begin{vmatrix} 4 & 0 \\ 2 & -1 \end{vmatrix} k$

$= 1\vec{i} + 2\vec{j} - 4\vec{k}$

$\vec{u} \cdot (\vec{u} \times \vec{v}) = 4(1) + 0(2) + 1(-4) = 0$

$\vec{v} \cdot (\vec{u} \times \vec{v}) = 2(1) + 2(-1) + 0(4) = 0$

10) $\vec{u} = -7i + 3j + k$, $\vec{v} = 2\vec{i} + 0\vec{j} + 4\vec{k}$

$\vec{u} \times \vec{v} = \begin{vmatrix} -3 & 1 \\ 0 & 4 \end{vmatrix} \vec{i} - \begin{vmatrix} -7 & 1 \\ 2 & 4 \end{vmatrix} \vec{j} + \begin{vmatrix} -7 & 3 \\ 2 & 0 \end{vmatrix} k = 12i + 30j - 6\vec{k}$

$\|12\vec{i} + 30\vec{j} - 6\vec{k}\| = \sqrt{144 + 900 + 36} = \sqrt{1080}$

$= \pm \left(\frac{12}{\sqrt{1080}} \vec{i} + \frac{30}{\sqrt{1080}} \vec{j} - \frac{6}{\sqrt{1080}} \vec{k} \right)$

$= \pm$

14) $\vec{u} = 2\vec{i} + 3\vec{j}$; $\vec{v} = -\vec{i} + 2\vec{j} - 2\vec{k}$

$$\vec{u} \times \vec{v} = \begin{vmatrix} 3 & 0 \\ 2 & -2 \end{vmatrix}\vec{i} - \begin{vmatrix} 2 & 0 \\ -1 & -2 \end{vmatrix}\vec{j} + \begin{vmatrix} 2 & 3 \\ -1 & 2 \end{vmatrix}\vec{k}$$

$$= -6\vec{i} + 4\vec{j} + 7\vec{k}$$

$$\|\vec{u} \times \vec{v}\| = \sqrt{36 + 16 + 49} = \underline{\underline{\sqrt{101}}}$$

23c) $\vec{u} = \langle 4, -8, 1 \rangle$, $\vec{v} = \langle 2, 1, -2 \rangle$, $\vec{w} = \langle 3, -4, 12 \rangle$

$$\vec{u} \cdot (\vec{v} \times \vec{w}) = \begin{vmatrix} 4 & -8 & 1 \\ 2 & 1 & -2 \\ 3 & -4 & 12 \end{vmatrix}$$

$$= 4\begin{vmatrix} 1 & -2 \\ -4 & 12 \end{vmatrix} + 8\begin{vmatrix} 2 & -2 \\ 3 & 12 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 3 & -4 \end{vmatrix}$$

$$= 4(12-8) + 8(24+6) + 1(-8-3)$$

$$= 4(4) + 8(30) + 1(-11)$$

$$= 16 + 240 - 11 = 245$$ <u>not coplanar</u>

30) $\vec{u} = 2\vec{i} + 3\vec{j} - 6\vec{k}$, $\vec{v} = 2\vec{i} + 3\vec{j} + 6\vec{k}$

(a) $\cos\theta = \dfrac{\vec{u} \cdot \vec{v}}{\|\vec{u}\|\|\vec{v}\|} = \dfrac{4 + 9 - 36}{(7)(7)} = \dfrac{-23}{49}$

(b) $\sin\theta = \dfrac{\|\vec{u} \times \vec{v}\|}{\|\vec{u}\|\|\vec{v}\|} = \dfrac{\|36\vec{i} - 24\vec{j}\|}{49} = \dfrac{12\sqrt{13}}{49}$

c) $\sin^2\theta + \cos^2\theta = 1$

$$\left(\frac{12\sqrt{13}}{49}\right)^2 + \left(\frac{-23}{49}\right)^2 = 1$$

Ch. 12

12.5

A line in 2- or 3-space can be uniquely determined by specifying a point on the line and a nonzero vector parallel to the line

Parametric eqns

A line in 2-space thru $P_0(x_0, y_0)$ $\parallel$ to nonzero vector $\vec{v} = \langle a, b \rangle$, has parametric eqns

$$x = x_0 + at, \; y = y_0 + bt \qquad (1)$$

A line in 3-space thru $P_0(x_0, y_0, z_0)$ $\parallel$ to nonzero vector $\vec{v} = \langle a, b, c \rangle$ has parametric eqns

$$x = x_0 + at, \; y = y_0 + bt, \; z = z_0 + ct \qquad (2)$$

Two lines in 3-space that are not parallel and do not intersect are called skew lines; any two skew lines lie in parallel planes.

Because two vectors are equal iff their components are equal, (1) & (2) can be written in vector form as

$$\langle x, y \rangle = \langle x_0 + at, \; y_0 + bt \rangle$$

$$\langle x, y, z \rangle = \langle x_0 + at, \; y_0 + bt, \; z_0 + ct \rangle$$

This form can be cleaned up and condensed to what is called the vector eqn of the line in 2-space or 3-space $\vec{r} = \vec{r}_0 + t\vec{v}$ where $\vec{v}$ is a nonzero vector $\parallel$ to the line & $\vec{r}_0$ is a vector whose components are the coordinates of a point on the line.

P. 829

4b) $P_1(-1,3,5)$, $P_2(-1,3,2)$

$\overrightarrow{P_1P_2} = \langle 0, 0, -3 \rangle$

$x = -1 + 0t, \; y = 3 + 0t, \; z = 5 - 3t$

for the segment, add condition $0 \le t \le 1$

5b) $x\vec{i} + y\vec{j} + z\vec{k} = \vec{k} + t(\vec{i} - \vec{j} + \vec{k})$

$P_0(0,0,1)$ $\vec{v} = \langle 1, -1, 1 \rangle$

$\therefore x = 0 + 1t, \; y = 0 - 1t, \; z = 1 + 1t$

9b) $x = 2 - t, \; y = -3 + 5t, \; z = t$

$\vec{r}_0 = \langle 2, -3, 0 \rangle, \; \vec{v} = \langle -1, 5, 1 \rangle$

$\vec{r} = \langle 2, -3, 0 \rangle + t\langle -1, 5, 1 \rangle$

$\vec{r} = 2\vec{i} - 3\vec{j} + t(-\vec{i} + 5\vec{j} + \vec{k})$

14) tangent to $y = x^2$ @ $(-2, 4)$

$y' = 2x = \frac{-4}{1}$

$\vec{v} = 1\vec{i} - 4\vec{j}$

$x = -2 + t, \; y = 4 - 4t$

P. 829

19) $x = 1+3t$, $y = 2-t$

a) x-axis ($y=0$)

$\therefore t = 2$

$x = 1+3(2)$

$x = 7$

b) y-axis ($x=0$)

$\therefore t = -\frac{1}{3}$

$y = 2-(-\frac{1}{3})$

$y = \frac{7}{3}$

c) $y = x^2$

$2-t = (1+3t)^2$

$2-t = 1+6t+9t^2$

$0 = -1+7t+9t^2$

$t = \frac{-7 \pm \sqrt{85}}{18}$

$x = 1+3t$

$x = \frac{-1 \pm \sqrt{85}}{6}$

$y = 2-t$

$y = \frac{43 \mp \sqrt{85}}{18}$

26) $L_1: x = -1+4t,\ y = 3+t,\ z = 1$

$L_2: x = -13+12t,\ y = 1+6t,\ z = 2+3t$

solve

$-1+4t_1 = -13+12t_2$ $\quad 3+t_1 = 1+6t_2$ $\quad 1 = 2+3t_2$

$4t_1 = -12-4$ $\quad t_1 = -2-2$ $\quad -\frac{1}{3} = t_2$

$4t_1 = -16$ $\quad t_1 = -4$ ✓

$t_1 = -4$ ✓

$\therefore x = -1+4(-4)$ $\quad y = 3-4$ $\quad z = 1$

$x = -17$ $\quad y = -1$

$(-17, -1, 1)$

28) $L_1: x = 2+8t,\ y = 6-8t,\ z = 10t$

$L_2: x = 3+8t,\ y = 5-3t,\ z = 6+t$

$\vec{v}_1 = \langle 8, -8, 10 \rangle,\ \vec{v}_2 = \langle 8, -3, 1 \rangle$

The vectors aren't ∥ so lines aren't ∥

$2+8t_1 = 3+8t_2,\ 6-8t_1 = 5-3t_2,\ 10t_1 = 6+t_2$

$6-8t_1 = 5-3t_2$

$8 = 8+5t_2$ $\quad 0 \neq 6$

$0 = t_2$

$\therefore$ lines don't intersect because the system of eqns has no soln

Since the lines are not ∥ & do not intersect they are skew

49) $L_1: x = 4-t,\ y = 1+2t,\ z = 2+t$

$L_2: x = t,\ y = 1+t,\ z = 1+2t$

a) $D = \sqrt{(0-4)^2+(1-1)^2+(1-2)^2} = \sqrt{17}$ cm

d) $D^2 = [t-(4-t)]^2 + [(1+t)-(1+2t)]^2 + [(1+2t)-(2+t)]^2$

$D^2 = (2t-4)^2 + (-t)^2 + (-1+t)^2$

$d(D^2) = 2(2t-4)(2) + 2t + 2(-1+t)$

$= 8t-16+2t-2+2t$

$0 = 12t-18$

$18 = 12t$

$3/2 = t$

$\therefore$ the minimum distance between the bugs is

$D = \sqrt{(-1)^2+(-\frac{3}{2})^2+(-1+\frac{3}{2})^2}$

$= \sqrt{1+\frac{9}{4}+\frac{1}{4}}$

$= \frac{\sqrt{14}}{2}$ cm

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Ch. 12

12.6

A plane in 3-space can be determined uniquely by specifying a point in the plane and a vector $\perp$ to the plane. A $\perp$ vector to a plane is called a normal to the plane

$$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$$ is the point-normal form of the eqn of a plane

$$\langle a,b,c\rangle \cdot \langle x-x_0, y-y_0, z-z_0\rangle = 0$$ is a vector version of this

If a, b, c, and d are constants and a, b, & c are not all zero, then the graph of the eqn

$$ax+by+cz+d=0 \qquad (6)$$

is a plane w/ $\vec{n}=\langle a,b,c\rangle$ as a normal

Eqn (6) is called the general form of the eqn of the plane

A unique plane is not determined by a point in the plane and a nonzero vector $\parallel$ to the plane

A unique plane is determined by a point in the plane and two non $\parallel$ vectors that are $\parallel$ to the plane

A unique plane is determined by 3 non collinear points that lie in the plane.

Two distinct intersecting planes determine two positive angles of intersection - an (acute) angle θ satisfying $0 \le \theta \le \pi/2$ and the supplement of that angle.

To find θ between the planes, $\cos\theta = \dfrac{|\vec{n}_1\cdot\vec{n}_2|}{\|\vec{n}_1\|\|\vec{n}_2\|}$ where $\vec{n}_1$ & $\vec{n}_2$ are the normals for the two planes

There are 3 basic distance problems in 3-space:

① Find the distance between a point & a plane

② Find the distance between 2 $\parallel$ planes

③ Find the distance between 2 skew lines

The distance D between a point $P(x_0, y_0, z_0)$ and the plane $ax+by+cz+d=0$ is

$$D=\frac{|ax_0+by_0+cz_0+d|}{\sqrt{a^2+b^2+c^2}}$$

P. 837

13b) $3x - 2y + z = 1$
$4x + 5y - 2z = 4$
$\langle 3, -2, 1 \rangle \cdot \langle 4, 5, -2 \rangle$
$12 - 10 - 2 = 0$
$\therefore \perp$

26) $P_1(-2, 1, 4)$, $P_2(1, 0, 3)$
$\perp$ $4x - y + 3z = 2$

$\overrightarrow{P_1P_2} = \langle 3, -1, -1 \rangle$
$\vec{n} = \langle 4, -1, 3 \rangle$

$$\vec{n} \times \overrightarrow{P_1P_2} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 4 & -1 & 3 \\ 3 & -1 & -1 \end{vmatrix}$$

$= 4\vec{i} + 13\vec{j} - 1\vec{k}$

$4(x+2) + 13(y-1) - 1(z-4) = 0$
$4x + 13y - 1z + 1 = 0$

37) $-2x + 3y + 7z + 2 = 0$
$x + 2y - 3z + 5 = 0$

$\vec{n}_1 = \langle -2, 3, 7 \rangle$

$\vec{n}_2 = \langle 1, 2, -3 \rangle$

$$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & 3 & 7 \\ 1 & 2 & -3 \end{vmatrix}$$

$\vec{v} = \langle -23, 1, -7 \rangle$ $\parallel$ to line of intersection

let $z = 0$, $-2x + 3y = -2$
$x + 2y = -8$
solve system
$x = -\frac{11}{7}$, $y = -\frac{12}{7}$, $z = 0$

$x = -\frac{11}{7} - 23t$, $y = -\frac{12}{7} + t$, $z = 0 - 7t$

44) $x = 3 - t$, $y = 4 + 4t$, $z = 1 + 2t$
$x = t$, $y = 3$, $z = 2t$

$(3, 4, 1)$, $(0, 3, 0)$

$\vec{v}_1 = \langle -1, 4, 2 \rangle$, $\vec{v}_2 = \langle 1, 0, 2 \rangle$

$\vec{v}_1 \times \vec{v}_2 = \langle 8, 4, -4 \rangle = 4\langle 2, 1, -1 \rangle$

so $2(x-0) + 1(y-3) - 1(z-0) = 0$
$2x + y - z - 3 = 0$ $\parallel L_1$, contains L_2

$$\therefore D = \frac{|2(3) + 1(4) - 1(1) - 3|}{\sqrt{4+1+1}}$$

$$= \frac{6}{\sqrt{6}} = \sqrt{6}$$

Ch. 12

12.7

Quadric Surfaces - the 3-dimensional analogs of the conic sections

A common method for building up the shape of a surface is w/ a network of mesh lines (curves obtained by cutting the surface w/ well chosen planes)

A mesh line that results when a surface is cut by a plane is called a trace of the surface in the plane

$Ax^2+By^2+Cz^2+Dxy+Exz+Fyz+Gx+Hy+Iz+J=0$ is called a second degree eqn in x, y, & z. These are called quadric surfaces or simply quadrics.

There are six nondegenerative types of quadrics:

Ellipsoids	$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	no minus signs	shaped like a football
Hyperboloid 1 sheet	$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$	one minus sign	nuclear plant cooling tower
Hyperboloid 2 sheets	$\frac{z^2}{c^2}-\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$	two minus signs	two symmetric bowls (separated)
Elliptic Cone	$z^2=\frac{x^2}{a^2}+\frac{y^2}{b^2}$	no linear term	two waffle cones point to point
Elliptic paraboloid	$z=\frac{x^2}{a^2}+\frac{y^2}{b^2}$	one linear term other both positive	one handle-less cup
Hyperbolic* paraboloid	$z=\frac{y^2}{b^2}-\frac{x^2}{a^2}$	one linear term other opposite signs	saddle

*The hyperbolic paraboloid has an interesting feature at the origin the xz trace has a relative max @ (0,0,0); the yz trace has a relative min @ (0,0,0)

A point w/ this property is called a saddle point or minimax point

Replacing a variable by its negative in the eqn of a surface causes that surface to be reflected @ a coordinate plane

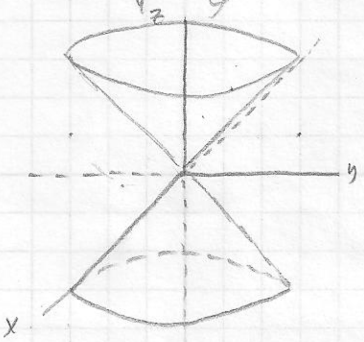
Interchanging two variables in the eqn of a surface reflects the surface about a plane that makes a 45° angle w/ two of the coordinate planes (it changes the axis of symmetry)

P.848

16) $9x^2+4y^2-36z^2=0$

$\frac{x^2}{4}+\frac{y^2}{9}-\frac{z^2}{1}=0$

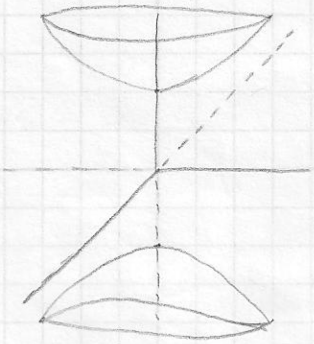
$z^2=\frac{x^2}{4}+\frac{y^2}{9}$ elliptic cone



17) $9z^2-4y^2-9x^2=36$

$\frac{z^2}{4}-\frac{y^2}{9}-\frac{x^2}{4}=1$ $\quad \frac{z^2}{4}-1=\frac{y^2}{9}+\frac{x^2}{4}$

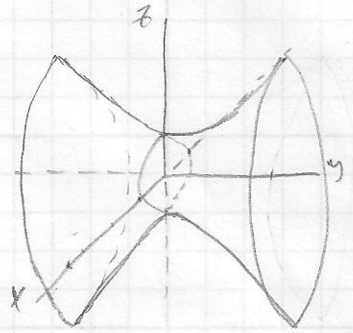
hyperboloid 2 sheets



20) $4x^2-y^2+4z^2=16$

$\frac{y^2}{16}+1=\frac{x^2}{4}+\frac{z^2}{4}$

hyperboloid of 1 sheet
symmetric to the y-axis



Ch. 12

12.8

Surfaces in Other Coordinate Systems

eq's in rectangular (x, y, z)

$x = x_0 \quad y = y_0 \quad z = z_0$

eq's in cylindrical (r, θ, z)

$r = r_0 \quad \theta = \theta_0 \quad z = z_0$

the surface $r = r_0$ is a right cylinder of radius r_0 centered on the z-axis
r has value r_0; θ & z are unrestricted except for general restriction $0 \le \theta < 2\pi$.

the surface $\theta = \theta_0$ is a half-plane attached along the z-axis & making an angle θ_0 w/ the positive x-axis; r & z are unrestricted except for general restriction that $r \ge 0$.

the surface $z = z_0$ is a horizontal plane; r & θ are unrestricted except for general restriction

eq's in spherical (ρ, θ, ϕ)

$\rho = \rho_0 \quad \theta = \theta_0 \quad \phi = \phi_0$

the surface $\rho = \rho_0$ consists of all points whose distance ρ from the origin is ρ_0 assuming $\rho_0 > 0$, this is a sphere of radius ρ_0 centered @ the origin.

the surface $\theta = \theta_0$ is the same as cylindrical

the surface $\phi = \phi_0$ consists of all points from which a line segment to the origin makes an angle of ϕ_0 w/ the positive z-axis. Depending on whether $0 < \phi_0 < \pi/2$ or $\pi/2 < \phi_0 < \pi$, this will be the nappe of a cone opening up or opening down

<u>Converting Coordinates</u>

cylindrical → rectangular	$x = r\cos\theta$	$y = r\sin\theta$	$z = z$
rectangular → cylindrical	$r = \sqrt{x^2+y^2}$	$\tan\theta = \frac{y}{x}$	$z = z$
spherical → rectangular	$x = \rho\cos\theta\sin\phi$	$y = \rho\sin\theta\sin\phi$	$z = \rho\cos\phi$
rectangular → spherical	$\rho = \sqrt{x^2+y^2+z^2}$	$\tan\theta = \frac{y}{x}$	$\cos\phi = \frac{z}{\rho}$
spherical → cylindrical	$r = \rho\sin\phi$	$\theta = \theta$	$z = \rho\cos\phi$
cylindrical → spherical	$\rho = \sqrt{r^2+z^2}$	$\theta = \theta$	$\tan\phi = \frac{r}{z}$

P. 854

2a) $(\sqrt{2}, -\sqrt{2}, 1)$

$r = \sqrt{(\sqrt{2})^2 + (-\sqrt{2})^2} = 2$

$\theta = \tan^{-1}(-1) = \frac{7\pi}{4}$

$z = 1$

$\underline{(2, \frac{7\pi}{4}, 1)}$

4d) $(4, \frac{\pi}{2}, -1)$

$x = 4\cos\frac{\pi}{2} = 0$

$y = 4\sin\frac{\pi}{2} = 4$

$z = -1$

$\underline{(0, 4, -1)}$

6c) $(2, 0, 0)$

$\rho = \sqrt{2^2 + 0^2 + 0^2} = 2$

$\tan\theta = \frac{0}{2} = 0$

$\phi = \cos^{-1}\frac{0}{2} = \frac{\pi}{2}$

$\underline{(2, 0, \frac{\pi}{2})}$

8b) $(3, \frac{7\pi}{4}, \frac{5\pi}{6})$

$x = 3\sin\frac{5\pi}{6}\cos\frac{7\pi}{4} = 3(\frac{1}{2})(\frac{\sqrt{2}}{2})$

$y = 3\sin\frac{5\pi}{6}\sin\frac{7\pi}{4} = 3(\frac{1}{2})(\frac{-\sqrt{2}}{2})$

$z = 3\cos\frac{5\pi}{6} = -\frac{3\sqrt{3}}{2}$

$\underline{(\frac{3\sqrt{2}}{4}, \frac{-3\sqrt{2}}{4}, -\frac{3\sqrt{3}}{2})}$

10c) $(4, \frac{\pi}{2}, 3)$

$\rho = \sqrt{4^2 + 3^2} = 5$

$\theta = \frac{\pi}{2}$

$\phi = \tan^{-1}(\frac{4}{3})$

$\underline{(5, \frac{\pi}{2}, \tan^{-1}(\frac{4}{3}))}$

12d) $(5, \frac{2\pi}{3}, \frac{5\pi}{6})$

$r = 5\sin\frac{5\pi}{6} = 5(\frac{1}{2}) = \frac{5}{2}$

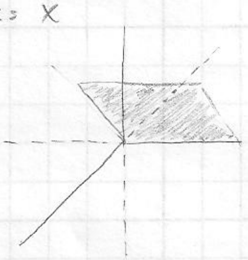
$\theta = \frac{2\pi}{3}$

$z = 5\cos\frac{5\pi}{6} = 5(\frac{-\sqrt{3}}{2}) = -\frac{5\sqrt{3}}{2}$

$\underline{(\frac{5}{2}, \frac{2\pi}{3}, \frac{-5\sqrt{3}}{2})}$

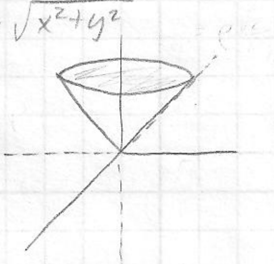
18) $z = r\cos\theta$

$z = x$



25) $\phi = \frac{\pi}{4}$

$z = \sqrt{x^2 + y^2}$



35) $x^2 + y^2 = 4$

(a) $\underline{r = 2}$

b) using $r = \rho\sin\phi$

$2 = \rho\sin\phi$

or

$\rho = 2\csc\phi$

38) $z^2 = x^2 - y^2$

$z^2 = r^2\cos^2\theta - r^2\sin^2\theta$

$z^2 = r^2(\cos^2\theta - \sin^2\theta)$

$z^2 = r^2\cos 2\theta$

b) $r = \rho\sin\phi$, $z = \rho\cos\phi$

$\rho^2\cos^2\phi = \rho^2\sin^2\phi\cos 2\theta$

$\cot^2\phi = \cos 2\theta$

44) $0 \le r \le 2\sin\theta$, $0 \le z \le 3$

right circular cylinder

height is 3, radius is 1

axis is line $x = 0, y = 1$

47) $(4000, 30°, 30°)$

$(4000, \frac{\pi}{6}, \frac{\pi}{6})$

$\underline{x = 4000\cos\frac{\pi}{6}\sin\frac{\pi}{6} = 1000\sqrt{3}}$

$y = 4000\sin\frac{\pi}{6}\sin\frac{\pi}{6} = 1000$

$z = 4000\cos\frac{\pi}{6} = 2000\sqrt{3}$

$(4000, \frac{\pi}{6}, \frac{\pi}{6})$

&

$\underline{(1000\sqrt{3}, 1000, 2000\sqrt{3})}$