

sec. 6.3 The Shell Method

1)  $V = 2\pi \int_a^b \rho(x) h(x) dx$

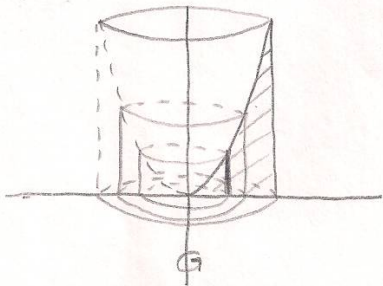
Vertical axis of rotation  
(plane parallel to y-axis)

2)  $V = 2\pi \int_c^d \rho(y) h(y) dy$

Horizontal axis of rotation  
(plane are parallel to x-axis)

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b)  $y = x^2, y = 0, x = 4, @ y\text{-axis}$



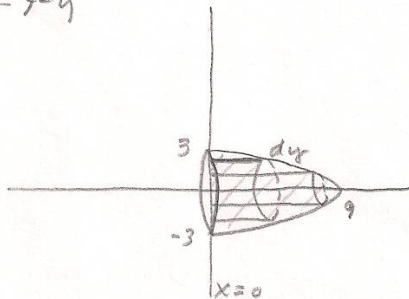
$$V = 2\pi \int_0^4 x(x^2) dx$$

$$V = 2\pi \int_0^4 x^3 dx$$

$$V = \frac{\pi x^4}{2} \Big|_0^4$$

$$V = \frac{256\pi}{2} = \underline{\underline{128\pi}}$$

1b)  $x + y^2 = 9, x = 0, @ x\text{-axis}$   
 $x = 9 - y^2$



$$V = 2\pi \int_0^3 y(9 - y^2) dy$$

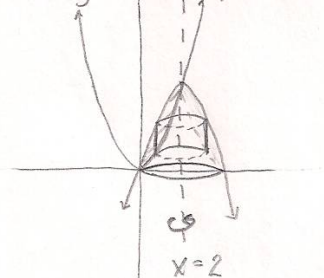
$$V = 2\pi \int_0^3 (9y - y^3) dy$$

$$V = \pi \left[ \frac{9}{2} y^2 - \frac{1}{4} y^4 \right] \Big|_0^3$$

$$V = \frac{\pi}{2} [18y^2 - y^4] \Big|_0^3$$

$$V = \frac{\pi}{2} [162 - 81] = \underline{\underline{\frac{81\pi}{2}}}$$

18)  $y = x^2, y = 4x - x^2, @ x = 2$



$$x^2 = 4x - x^2$$

$$2x^2 - 4x = 0$$

$$2x(x - 2) = 0$$

$$x = 0, 2$$

$$V = 2\pi \int_0^2 (2-x)(4x - x^2 - x^2) dx$$

$$V = 2\pi \int_0^2 (2-x)(4x - 2x^2) dx$$

$$V = 2\pi \int_0^2 (8x - 8x^2 + 2x^3) dx$$

$$V = 2\pi \left[ 4x^2 - \frac{8}{3}x^3 + \frac{1}{2}x^4 \right] \Big|_0^2$$

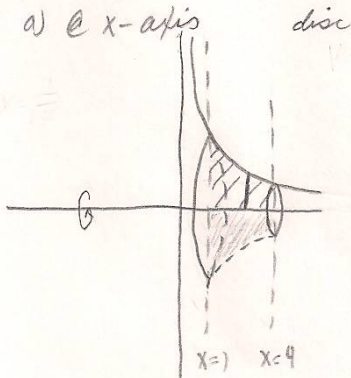
$$V = 2\pi \left[ 16 - \frac{64}{3} + 8 \right]$$

$$V = 2\pi \left[ 24 - \frac{64}{3} \right]$$

$$V = \underline{\underline{\frac{16}{3}\pi}}$$

22)  $y = \frac{1}{x^2}$ ,  $y=0$ ,  $x=1$ ,  $x=4$

a) @  $x$ -axis



disc

$$V = \pi \int_1^4 (x^{-2})^2 dx$$

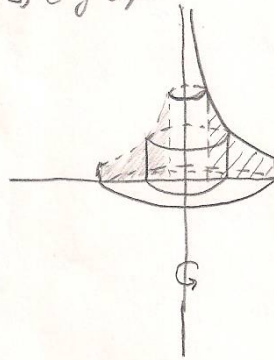
$$V = \pi \int_1^4 x^{-4} dx$$

$$V = \pi \left( -\frac{1}{3x^3} \right) \Big|_1^4$$

$$V = \pi \left[ -\frac{1}{192} + \frac{1}{3} \right]$$

$$V = \underline{\underline{\frac{21\pi}{64}}}$$

b) @  $y$ -axis



$$V = 2\pi \int_1^4 x(x^{-2}) dx$$

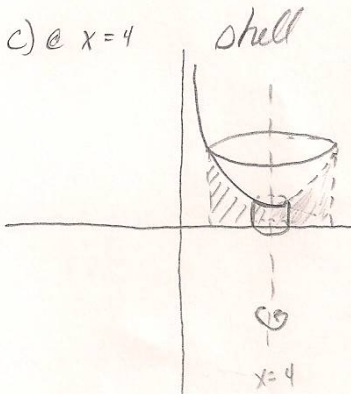
$$V = 2\pi \int_1^4 \frac{1}{x} dx$$

$$V = 2\pi [\ln x] \Big|_1^4$$

$$V = 2\pi [\ln 4 - \ln 1]$$

$$V = \underline{\underline{2\pi \ln 4}}$$

c) @  $x=4$



shell

$$V = 2\pi \int_1^4 (4-x)x^{-2} dx$$

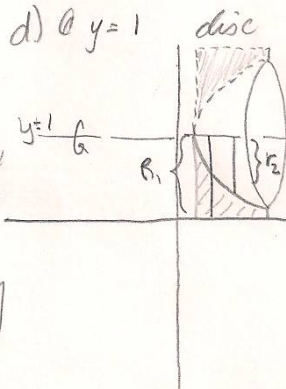
$$V = 2\pi \int_1^4 \left( 4x^{-2} - \frac{1}{x} \right) dx$$

$$V = 2\pi \left[ -\frac{4}{x} - \ln x \right] \Big|_1^4$$

$$V = 2\pi [-1 - \ln 4 + 4 + 0]$$

$$V = 2\pi (3 - \ln 4)$$

d) @  $y=1$



disc

$$V = \pi \int_1^4 \left( (1)^2 - \left(1 - \frac{1}{x^2}\right)^2 \right) dx$$

$$V = \pi \int_1^4 \left( 1 - 1 + \frac{2}{x^2} - \frac{1}{x^4} \right) dx$$

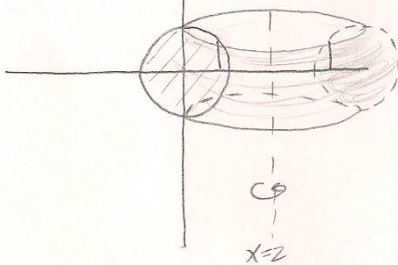
$$V = \pi \left[ -\frac{2}{x} + \frac{1}{3x^3} \right] \Big|_1^4$$

$$V = \pi \left[ -\frac{1}{2} + \frac{1}{192} + 2 - \frac{1}{3} \right]$$

$$V = \underline{\underline{\frac{75}{64}\pi}}$$

30)  $x^2 + y^2 = 1$ , @  $x=2$

$$\int_1^2 \sqrt{1-x^2} dx$$



$$V = 2 \cdot 2\pi \int_{-1}^1 (2-x)(\sqrt{1-x^2}) dx$$

$$V = 4\pi \int_{-1}^1 (2\sqrt{1-x^2} - x\sqrt{1-x^2}) dx$$

$$V = 4\pi$$