sec 6.2 Volume: Disc Method

$V = \pi \int_a^b [R(x)]^2 \, dx$ horizontal axis of revolution (slice is perpendicular to axis)

$V = \pi \int_c^d [R(y)]^2 \, dy$ vertical axis of revolution (slice is perpendicular to axis)

Washer Method (disc w/a hole)

$V = \pi \int_a^b [R(x)]^2 - [r(x)]^2 \, dx$ Outer radius - Inner radius

Volumes of Solids w/Known cross section

1) cross section $\perp$ x-axis

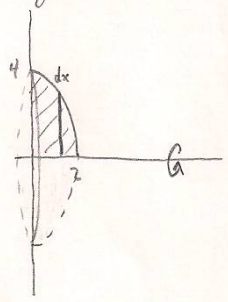
$V = \int_a^b A(x) \, dx$ $A(x)$ is area of cross section

2) cross section $\perp$ y-axis

$V = \int_a^b A(y) \, dy$ $A(y)$ is area of cross section

P. 420

2) $y = 4 - x^2$



$V = \pi \int_0^2 (4 - x^2)^2 \, dx$

$V = \pi \int_0^2 16 - 8x^2 + x^4 \, dx$

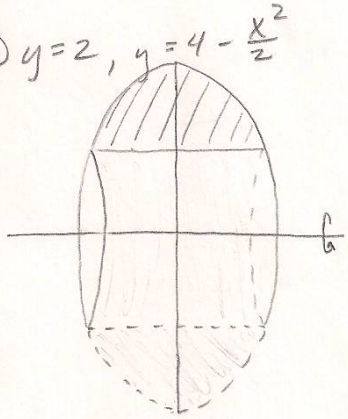
$V = \pi \left[16x - \frac{8}{3}x^3 + \frac{1}{5}x^5\right]_0^2$

$V = \pi \left[32 - \frac{64}{3} + \frac{32}{5}\right]$

$V = \pi \left[32 - \frac{224}{15}\right]$

$V = \frac{256}{15}\pi$

8) $y = 2$, $y = 4 - \frac{x^2}{2}$



$V = 2\pi \int_0^2 \left(4 - \frac{x^2}{2}\right)^2 - (2)^2 \, dx$

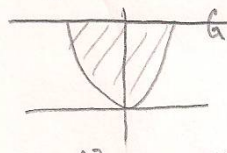
$V = 2\pi \int_0^2 16 - 4x^2 + \frac{x^4}{4} - 4 \, dx$

$V = 2\pi \int_0^2 12 - 4x^2 + \frac{x^4}{4} \, dx$

$V = 2\pi \left[12x - \frac{4x^3}{3} + \frac{x^5}{20}\right]_0^2$

$V = 2\pi \left[24 - \frac{32}{3} + \frac{32}{20}\right] = \frac{448}{15}\pi$

18) $y = x^2$, $y = 4$ @ $y = 4$



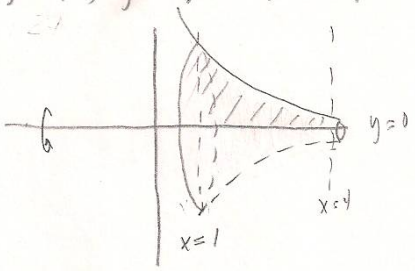
$V = 2\pi \int_0^2 (4 - x^2)^2 \, dx$

$V = 2\pi \int_0^2 16 - 8x^2 + x^4 \, dx$

$V = 2\pi \left[16x - \frac{8}{3}x^3 + \frac{1}{5}x^5\right]_0^2$

$V = 2\pi \left[32 - \frac{64}{3} + \frac{32}{5}\right] = \frac{512}{15}\pi$

27) $y = \frac{1}{x}$, $y = 0$, $x = 1$, $x = 4$, @ x-axis

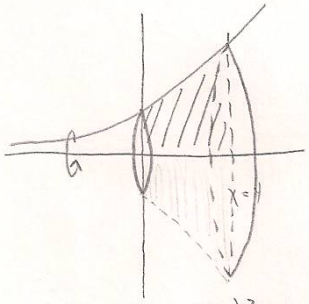


$V = \pi \int_1^4 \left(\frac{1}{x}\right)^2 dx$

$V = \pi \int_1^4 x^{-2} \, dx$

$V = \pi \left[-\frac{1}{x}\right]\Big|_1^4 = \pi\left[-\frac{1}{4} + 1\right] = \frac{3}{4}\pi$

30) $y = e^{x/2}$, $y = 0$, $x = 0$, $x = 4$, @ x-axis



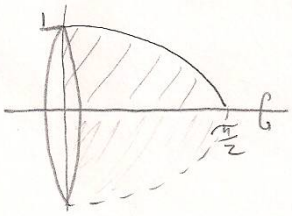
$V = \pi \int_0^4 (e^{x/2})^2 \, dx$

$V = \pi \int_0^4 e^x \, dx$

$V = \pi e^x \Big|_0^4$

$V = \pi(e^4 - 1)$

32) $y = \sqrt{\cos x}$, $y = 0$, $x = 0$, $x = \frac{\pi}{2}$, @ x-axis



$V = \pi \int_0^{\pi/2} (\sqrt{\cos x})^2 \, dx$

$V = \pi \int_0^{\pi/2} \cos x \, dx$

$V = \pi \sin x \Big|_0^{\pi/2}$

$V = \pi$