Sec. 7.4 Trigonometric Substitution

The objective is to replace the radical in the integrand by use of Pythagorean identities:

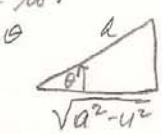
$\cos^2\theta = 1 - \sin^2\theta$

$\sec^2\theta = 1 + \tan^2\theta$

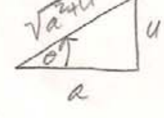
$\tan^2\theta = 1 - \sec^2\theta$

Three basic cases to try to fit your problem to:

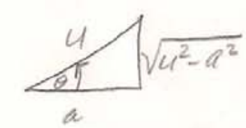
1) If integrand has $\sqrt{a^2-u^2}$, let $u = a\sin\theta$



2) If integrand has $\sqrt{a^2+u^2}$, let $u = a\tan\theta$



3) If integrand has $\sqrt{u^2-a^2}$, let $u = a\sec\theta$



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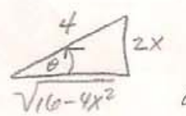
22) $\int_0^2 x\sqrt{16-4x^2}\,dx$



$\sin\theta = \frac{2x}{4}$

$\cos\theta\, d\theta = \frac{1}{2}dx$

$\cos\theta = \frac{\sqrt{16-4x^2}}{4}$

$\int 2\sin\theta\, 4\cos\theta\, 2\cos\theta\, d\theta$

$16\int \sin\theta\cos^2\theta\, d\theta \quad u = \cos\theta$

$du = -\sin\theta\, d\theta$

$-16\int u^2\, du$

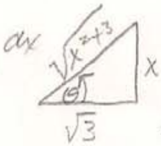
$-\frac{16}{3}u^3 + c$

$-\frac{16}{3}\cos^3\theta + c$

$-\frac{16}{3}\left(\frac{\sqrt{16-4x^2}}{4}\right)^3 \Big|_0^2$

$-\frac{16}{3}(0-1) = \frac{16}{3}$

28) $\int \frac{1}{(x^2+3)^{3/2}}\,dx$



$\tan\theta = \frac{x}{\sqrt{3}}$

$\sec\theta = \frac{\sqrt{x^2+3}}{\sqrt{3}}$

$\sec^2\theta\, d\theta = \frac{1}{\sqrt{3}}dx$

$\int \frac{1}{(\sqrt{3}\sec\theta)^3}\cdot\sqrt{3}\sec^2\theta\, d\theta$

$\frac{1}{3}\int \frac{\sec^2\theta}{\sec^3\theta}\, d\theta$

$\frac{1}{3}\int \frac{1}{\sec\theta}\, d\theta$

$\frac{1}{3}\int \cos\theta\, d\theta$

$\frac{1}{3}\sin\theta + C$

$\frac{1}{3}\frac{x}{\sqrt{x^2+3}} + C$