## Sec 7.3 Trigonometric Integrals

You'll be studying techniques for evaluating integrals of the form $\int \sin^m x \cos^n x\,dx$ & $\int \sec^m x \tan^n x\,dx$. Identities will be critical here.

Form 1 $\int \sin^m x \cos^n x\,dx$

case 1: at least one of m & n is an odd positive integer. If so, the other can be $\in \mathbb{R}$

case 2: Both m & n are non-negative even integers

Remember these identities: $\sin^2 x + \cos^2 x = 1$

$\sin^2 x = \frac{1}{2}(1-\cos 2x)$

$\cos^2 x = \frac{1}{2}(1+\cos 2x)$

Form 2 $\int \sec^m x \tan^n x\,dx$

case 1: m is an even positive integer

case 2: n is an odd positive integer

Remember: $\tan^2 x + 1 = \sec^2 x$

Bear in mind when you're manipulating your integral that you need to have something left over for du.

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2) $\int \cos^3 x \sin x\,dx$ Since $D_x \sin x = \cos x$, pull a $\cos x$ off of $\cos^3 x$ for du, convert the $\cos^2 x$ into $1-\sin^2 x$

$\int (1-\sin^2 x)\sin x \cos x\,dx$

$\int \sin x \cos x - \sin^3 x \cos x\,dx$ $\quad u = \sin x$

$\int u - u^3\,du$ $\quad du = \cos x\,dx$

$\frac{1}{2}u^2 - \frac{1}{4}u^4 + c$

$\frac{1}{2}\sin^2 x - \frac{1}{4}\sin^4 x + C$

10) $\int x^2 \sin^2 x\,dx$

$\int x^2 \frac{1}{2}(1-\cos 2x)\,dx$ Now integrate by parts

| | u | dv |
|---|---|---|
| + | $x^2$ | $\frac{1}{2}(1-\cos 2x)$ |
| - | $2x$ | $\frac{1}{2}(x-\frac{1}{2}\sin 2x)$ |
| + | $2$ | $\frac{1}{2}(\frac{1}{2}x^2+\frac{1}{4}\cos 2x)$ |
| 0 | 0 | $\frac{1}{2}(\frac{1}{6}x^3+\frac{1}{8}\sin 2x)$ |

$\frac{1}{2}x^2(x-\frac{1}{2}\sin 2x) - x(\frac{1}{2}x^2+\frac{1}{4}\cos 2x) + (\frac{1}{6}x^3+\frac{1}{8}\sin 2x) + C$

$\cancel{\frac{1}{2}x^3} - \frac{1}{4}x^2\sin 2x - \cancel{\frac{1}{2}x^3} - \frac{1}{4}x\cos 2x + \frac{1}{6}x^3 + \frac{1}{8}\sin 2x + C = \frac{1}{6}x^3 - \frac{1}{4}x^2\sin 2x - \frac{1}{4}x\cos 2x + \frac{1}{8}\sin 2x + C$

18) $\int \sec^6 \frac{x}{2}\,dx$ Since $D_x \tan x = \sec^2 x$, break apart $\sec^6 \frac{x}{2}$ into $\sec^4 \frac{x}{2} + \sec^2 \frac{x}{2}$. Then rewrite $\sec^4 \frac{x}{2}$ in term of tan.

$\int \sec^4 \frac{x}{2} \sec^2 \frac{x}{2}\,dx$

$\int [\sec^2 \frac{x}{2}]^2 \sec^2 \frac{x}{2}\,dx$

$\int [\tan^2 \frac{x}{2} + 1]^2 \sec^2 \frac{x}{2}\,dx$ f.o.i.l. out your binomial

$\int [\tan^4 \frac{x}{2} + 2\tan^2 \frac{x}{2} + 1] \sec^2 x\,dx$ distribute thru your parenthesis

$\int \tan^4 \frac{x}{2} \sec^2 \frac{x}{2} + 2\tan^2 \frac{x}{2} \sec^2 \frac{x}{2} + \sec^2 \frac{x}{2}\,dx$ Substitute as needed & integrate

$u = \tan \frac{x}{2}$ $\quad$ $u = \tan \frac{x}{2}$

$du = \frac{1}{2}\sec^2 \frac{x}{2}$ $\quad$ $du = \frac{1}{2}\sec^2 \frac{x}{2}$

$2\int u^4\,du + 4\int u^2\,du + \int \sec^2 \frac{x}{2}\,dx$

$\frac{2}{5}u^5 + \frac{4}{3}u^3 + \tan \frac{x}{2} + C$

$\frac{2}{5}\tan^5 \frac{x}{2} + \frac{4}{3}\tan^3 \frac{x}{2} + \tan \frac{x}{2} + C$

42) $\int \csc^2 3x \cot 3x\,dx$ $D_x \csc u = \csc u \cot u\,du$ $\therefore$

$\int \csc 3x \,\csc 3x \cot 3x\,dx$ $\quad$ $u = \csc 3x$

$du = 3\csc 3x \cot 3x\,dx$

$\frac{1}{3}\int u\,du$

$\frac{1}{3} \cdot \frac{1}{2}u^2 + C$

$\frac{1}{6}\csc^2 3x + C$

54) $\int_{-\pi}^{\pi} \sin 3\theta \cos\theta\,d\theta$ Using the second identity listed at the top of page 496

$\sin 3\theta \cos\theta = \frac{1}{2}[\sin(3-1)x + \sin(3+1)x]$

$\sin 3\theta \cos\theta = \frac{1}{2}[\sin 2x + \sin 4x]$

$\frac{1}{2}\int_{-\pi}^{\pi} \sin 2x + \sin 4x\,dx$

$u = 2x$ $\quad$ $u = 4$

$\frac{1}{2}\left[\frac{1}{2}\cos 2x - \frac{1}{4}\cos 4x\right]\Big|_{-\pi}^{\pi}$

$\frac{1}{2}\left[\left(-\frac{1}{2}\cos 2\pi - \frac{1}{4}\cos 4\pi\right) - \left(-\frac{1}{2}\cos(-2\pi) - \frac{1}{4}\cos(-4\pi)\right)\right]$

$\frac{1}{2}\left[-\frac{1}{2} - \frac{1}{4} + \frac{1}{2} + \frac{1}{4}\right] = 0$