

### Summary of Convergence Tests

NAME	STATEMENT	COMMENTS
<b>Divergence Test</b> (11.4.1)	If $\lim_{k \rightarrow +\infty} u_k \neq 0$ , then $\sum u_k$ diverges.	If $\lim_{k \rightarrow +\infty} u_k = 0$ , then $\sum u_k$ may or may not converge.
<b>Integral Test</b> (11.4.4)	Let $\sum u_k$ be a series with positive terms, and let $f(x)$ be the function that results when $k$ is replaced by $x$ in the general term of the series. If $f$ is decreasing and continuous for $x \geq 1$ , then $\sum_{k=1}^{\infty} u_k \quad \text{and} \quad \int_1^{+\infty} f(x) dx$ both converge or both diverge.	This test only applies to series that have positive terms.  Try this test when $f(x)$ is easy to integrate.
<b>Comparison Test</b> (11.6.1)	Let $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$ be series with nonnegative terms such that $a_1 \leq b_1, a_2 \leq b_2, \dots, a_k \leq b_k, \dots$ If $\sum b_k$ converges, then $\sum a_k$ converges, and if $\sum a_k$ diverges, then $\sum b_k$ diverges.	This test only applies to series with nonnegative terms.  Try this test as a last resort; other tests are often easier to apply.
<b>Ratio Test</b> (11.6.5)	Let $\sum u_k$ be a series with positive terms and suppose that $\rho = \lim_{k \rightarrow +\infty} \frac{u_{k+1}}{u_k}$ (a) Series converges if $\rho < 1$ . (b) Series diverges if $\rho > 1$ or $\rho = +\infty$ . (c) The test is inconclusive if $\rho = 1$ .	Try this test when $u_k$ involves factorials or $k$ th powers.
<b>Root Test</b> (11.6.6)	Let $\sum u_k$ be a series with positive terms such that $\rho = \lim_{k \rightarrow +\infty} \sqrt[k]{u_k}$ (a) The series converges if $\rho < 1$ . (b) The series diverges if $\rho > 1$ or $\rho = +\infty$ . (c) The test is inconclusive if $\rho = 1$ .	Try this test when $u_k$ involves $k$ th powers.
<b>Limit Comparison Test</b> (11.6.4)	Let $\sum a_k$ and $\sum b_k$ be series with positive terms such that $\rho = \lim_{k \rightarrow +\infty} \frac{a_k}{b_k}$ If $0 < \rho < +\infty$ , then both series converge or both diverge.	This is easier to apply than the comparison test, but still requires some skill in choosing the series $\sum b_k$ for comparison.
<b>Alternating Series Test</b> (11.7.1)	If $a_k > 0$ for $k = 1, 2, 3, \dots$ , then the series $a_1 - a_2 + a_3 - a_4 + \dots$ $-a_1 + a_2 - a_3 + a_4 - \dots$ converge if the following conditions hold: (a) $a_1 > a_2 > a_3 > \dots$ (b) $\lim_{k \rightarrow +\infty} a_k = 0$	This test applies only to alternating series.
<b>Ratio Test for Absolute Convergence</b> (11.7.5)	Let $\sum u_k$ be a series with nonzero terms such that $\rho = \lim_{k \rightarrow +\infty} \frac{ u_{k+1} }{ u_k }$ (a) The series converges absolutely if $\rho < 1$ . (b) The series diverges absolutely if $\rho > 1$ or $\rho = +\infty$ . (c) The test is inconclusive if $\rho = 1$ .	The series need not have positive terms and need not be alternating to use this test.