sec. 7.5 Partial Fractions

The objective is to break down an involved rational integrand into smaller, more easily integrated fractions.

Steps $R(x) = \frac{N(x)}{D(x)}$; $D(x)$ comprised of linear factors

1) If $\deg N(x) \geq \deg D(x)$, divide and then work on breaking down remainder

2) Factor $D(x)$ completely
   a) If there are repeated factors, run them down thru the exponents to one.
   b) Set $\dfrac{N(x)}{D(x)[\text{factored}]} = \dfrac{A}{1^{st}\text{ factor}} + \dfrac{B}{2^{nd}\text{ factor}} + \ldots$

3) Multiply thru by the LCD.

4) Choose values of x to eliminate individual coefficients; equate coefficients if the first half fails.

5) Place the coefficients you have found in place in the numerators from (2b).

6) Integrate the simpler partial fractions.

If you have non-factorable quadratic factors, set up is the same as for linear factors except the coefficients are of the form $Bx+C$
[A short-cut: to save trouble in the integrating, use the derivative of the factor to set up $Bx+C$]

P. 517

8) $\displaystyle\int \frac{1}{4x^2-9}\,dx$

$$\frac{1}{4x^2-9} = \frac{1}{(2x-3)(2x+3)} = \frac{A}{2x-3} + \frac{B}{2x+3}$$

$1 = A(2x+3) + B(2x-3)$

set $x = \frac{3}{2}$ $\quad 1 = 6A$

$\frac{1}{6} = A$

set $x = -\frac{3}{2}$ $\quad 1 = -6B$

$-\frac{1}{6} = B$

$$\therefore \int \frac{1}{4x^2-9}\,dx = \int \frac{\frac{1}{6}}{2x-3} + \frac{-\frac{1}{6}}{2x+3}\,dx$$

$$= \frac{1}{6}\int \frac{1}{2x-3}\,dx - \frac{1}{6}\int \frac{1}{2x+3}\,dx$$

$u = 2x-3$
$du = 2\,dx$

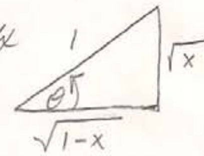
$w = 2x+3$
$dw = 2\,dx$

$$= \frac{1}{12}\int \frac{1}{u}\,du - \frac{1}{12}\int \frac{1}{w}\,dw$$

$$= \frac{1}{12}\ln|2x-3| - \frac{1}{12}\ln|2x+3| + C$$

$$= \frac{1}{12}\ln\left|\frac{2x-3}{2x+3}\right| + C$$

34) $\int \frac{\sqrt{1-x}}{\sqrt{x}}\,dx$



$\sin\theta = \sqrt{x}$

$\cos\theta = \sqrt{1-x}$

$\cos\theta\, d\theta = \frac{1}{2\sqrt{x}}\,dx$

$\int \cos\theta\, 2\cos\theta\, d\theta$

$2\int \cos^2\theta\, d\theta$

$2\int \frac{1}{2}(1+\cos 2\theta)\, d\theta$

$\int 1 + \cos 2\theta \;\; d\theta$

$\theta + \frac{1}{2}\sin 2\theta + C$

$\sin^{-1}\sqrt{x} + \frac{1}{2}(2\sin\theta\cos\theta) + c$

$\sin^{-1}\sqrt{x} + \sqrt{x}\,\sqrt{1-x} + c$

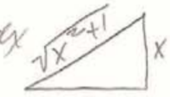
36) $\int \frac{x^3+x+1}{x^4+2x^2+1}\,dx$

$u = x^4+2x^2+1$

$du = 4x^3+4x \;\; dx$

$\int \frac{x^3+x}{x^4+2x^2+1}\,dx + \int \frac{1}{x^4+2x^2+1}\,dx$

$\frac{1}{4}\int \frac{1}{u}\,du + \int \frac{1}{(x^2+1)^2}\,dx$



$\frac{1}{4}\ln u + \int \frac{\sec^2\theta\, d\theta}{\sec^4\theta}$

$\tan\theta = x$

$\sec\theta = \sqrt{x^2+1}$

$\sec^2\theta\, d\theta = dx$

$\frac{1}{4}\ln(x^4+2x^2+1) + \int \frac{1}{\cos^2\theta}\,d\theta$

$\frac{1}{4}\ln(x^4+2x^2+1) + \int \sec^2\theta\, d\theta$

$\frac{1}{4}\ln(x^4+2x^2+1) + \tan\theta + C$

$\frac{1}{4}\ln(x^4+2x^2+1) + x + C$