sec. 7.2 Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

Two principles to bear in mind:
1) $v = \int dv$ is easy to find
2) $\int v\,du$ is easier to compute than $u\,dv$

A few Rules of Thumb to keep track of when trying to choose u & dv:
1) ln is always u.
2) if ln is not present, choose integer powers of x to be u so they will eventually differentiate to zero.
3) e^u can always be dv because they are easy to integrate

Trig functions almost always oscillate so you have to integrate twice and then "resolve" for your integral.

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10) $\int x e^{-x}\,dx$ $\quad u = x \quad dv = e^{-x}\,dx$
$\quad du = dx \quad v = -e^{-x}$

$-xe^{-x} - \int -e^{-x}\,dx$

$-xe^{-x} + \int e^{-x}\,dx$

$-xe^{-x} - e^{-x} + C$

14) $\int x^3 \ln x\,dx$ $\quad u = \ln x \quad dv = x^3\,dx$
$\quad du = \frac{1}{x}\,dx \quad v = \frac{1}{4}x^4$

$\frac{1}{4}x^4 \ln x - \int \frac{1}{4}x^4 \cdot \frac{1}{x}\,dx$

$\frac{1}{4}x^4 \ln x - \frac{1}{4}\int x^3\,dx$

$\frac{1}{4}x^4 \ln x - \frac{1}{4}\cdot\frac{1}{4}x^4 + C$

$\frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + C$

20) $\int \frac{x^3 e^{x^2}}{(x^2+1)^2}\,dx$ $\quad u = x^2 e^{x^2} \quad dv = \frac{x}{(x^2+1)^2}\,dx$
$\quad du = 2xe^{x^2} + 2x^3 e^{x^2} \quad v = \frac{-1}{2(x^2+1)}$

$-\frac{x^2 e^{x^2}}{2(x^2+1)} - \int 2xe^{x^2}(x^2+1)\cdot\frac{-1}{2(x^2+1)}\,dx$

$-\frac{x^2 e^{x^2}}{2(x^2+1)} + \int x e^{x^2}\,dx$ $\quad u = x^2$
$\quad du = 2x\,dx$

$-\frac{x^2 e^{x^2}}{2(x^2+1)} + \frac{1}{2}e^{x^2} + C$

26) $\int \theta \sec\theta \tan\theta\,d\theta$ $\quad u = \theta \quad dv = \sec\theta\tan\theta\,d\theta$
$\quad du = d\theta \quad v = \sec\theta$

$\theta\sec\theta - \int \sec\theta\,d\theta$

$\theta\sec\theta - \ln|\sec\theta + \tan\theta|$

28) $\int \cos^{-1}x\,dx$ $\quad u = \cos^{-1}x \quad dv = dx$
$\quad du = \frac{-1}{\sqrt{1-x^2}}\,dx \quad v = x$

$x\cos^{-1}x - \int \frac{-x}{\sqrt{1-x^2}}\,dx$ $\quad u = 1-x^2$
$\quad du = -2x\,dx$

$x\cos^{-1}x - \frac{1}{2}\int u^{-1/2}\,du$

$x\cos^{-1}x - \sqrt{1-x^2} + C$

Integration by Parts: Tabular Method

Signs	Derivatives u	(multiply)	Integrals dv
+	___	$\searrow$	___
−	___	$\searrow$	___
+	___		___
⋮	⋮		⋮
	___	$\searrow$	___
+	___	$\rightarrow$	___

Stop when the horizontal product is:
a) zero
b) integrable
c) same as the original integrand

Functions that work the best on table:
$\int x^n \sin ax\,dx$, $\int x^n \cos ax\,dx$, or $\int x^n e^{ax}dx$

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11) $\int x^3 e^x\,dx$

	u	dv
+	x^3	e^x
−	$3x^2$	e^x
+	$6x$	e^x
−	6	e^x
+	0	e^x

$\underline{\underline{x^3e^x - 3x^2e^x + 6xe^x - 6e^x + C}}$

30) $\int e^x \cos 2x\,dx$

	u	dv
+	$\cos 2x$	e^x
−	$-2\sin 2x$	e^x
+	$4\cos 2x$	e^x

$\int e^x \cos 2x\,dx = e^x\cos 2x + 2e^x\sin 2x + 4\int e^x\cos 2x\,dx$

$-3\int e^x\cos 2x\,dx = e^x\cos 2x + 2e^x \sin 2x$

$\int e^x \cos 2x\,dx = \underline{\underline{-\frac{1}{3}\left[e^x\cos 2x + 2e^x\sin 2x\right] + C}}$

40) $\int_0^1 x^2 e^x\,dx$

	u	dv
+	x^2	e^x
−	$2x$	e^x
+	2	e^x
−	0	e^x

$x^2e^x - 2xe^x + 2e^x \Big|_0^1$

$(e - 2e + 2e) - (0 - 0 + 2)$

$\underline{\underline{e - 2}}$

46) $\int x^3 \cos 2x\,dx$

	u	dv
+	x^3	$\cos 2x$
−	$3x^2$	$-2\sin 2x$
+	$6x$	$-4\cos 2x$
−	6	$8\sin 2x$
+	0	$16\cos 2x$

$\underline{\underline{-2x^3\sin 2x + 12x^2\cos^2 x + 48x\sin 2x - 96\cos 2x + C}}$

74) $y = e^{-4t}(\cos 2t + 5\sin 2t)$ $[0, \pi]$ avg value $= \frac{1}{b-a}\int f(x)\,dx$

$\frac{1}{\pi}\int_0^{\pi} e^{-4t}(\cos 2t + 5\sin 2t)\,dt$

$\frac{1}{\pi}\int_0^{\pi} e^{-4t}\cos 2t\,dt + \frac{5}{\pi}\int_0^{\pi} e^{-4t}\sin 2t\,dt$

	u	dv		u	dv
+	$\frac{1}{\pi}\cos 2t$	e^{-4t}	+	$\frac{5}{\pi}\sin 2t$	e^{-4t}
−	$-\frac{2}{\pi}\sin 2t$	$-\frac{1}{4}e^{-4t}$	−	$\frac{10}{\pi}\cos 2t$	$-\frac{1}{4}e^{-4t}$
+	$-\frac{4}{\pi}\cos 2t$	$\frac{1}{16}e^{-4t}$	+	$-\frac{20}{\pi}\sin 2t$	$\frac{1}{16}e^{-4t}$

$\frac{1}{\pi}\int_0^{\pi} e^{-4t}\cos 2t\,dt + \frac{5}{\pi}\int_0^{\pi}\sin 2t\,dt =$

$-\frac{1}{4\pi}e^{-4t}\cos 2t - \frac{1}{2\pi}e^{-4t}\sin 2t - \int_0^{\pi}\frac{1}{4\pi}e^{-4}\cos 2t\,dt$

$-\frac{5}{4\pi}e^{-4t}\sin 2t - \frac{5}{8\pi}e^{-4t}\cos 2t - \int_0^{\pi}\frac{5}{4\pi}\sin 2t\,dt$

$\frac{5}{4\pi}\int_0^{\pi} e^{-4t}\cos 2t\,dt + \frac{25}{4\pi}\int_0^{\pi} e^{-4t}\sin 2t\,dt = -\frac{7}{8\pi}e^{-4t}\cos 2t - \frac{7}{4\pi}e^{-4t}\sin 2t$

$\int_0^{\pi} e^{-4t}\cos 2t\,dt + \int_0^{\pi} e^{-4t}\sin 2t\,dt = -\frac{7}{10}e^{-4t}\cos 2t - \frac{7}{2\pi}e^{-4t}\sin 2t \Big|_0^{\pi}$

$= -\frac{7}{10}e^{-4\pi} + \frac{7}{10} = \frac{7}{10}(1 - e^{-4\pi})$