Ch. 6 Applications of Integration

General formulas for finding the area between two curves:

$A = \int_{x_1}^{x_2} (\text{Top} - \text{Bottom})\, dx$ slices are vertical

$A = \int_{y_1}^{y_2} (\text{Right} - \text{Left})\, dy$ slices are horizontal

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4) $f(x) = x^2$, $g(x) = x^3$

$x^2 = x^3$

$x^2 - x^3 = 0$

$x^2(1-x) = 0$

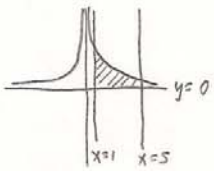
$x = 0, x = 1$

$A = \int_0^1 x^2 - x^3\, dx$

$A = \frac{1}{3}x^3 - \frac{1}{4}x^4 \Big|_0^1$

$A = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$

18) $y = \frac{1}{x^2}$, $y = 0$, $x = 1$, $x = 5$

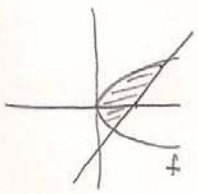


$A = \int_1^5 \frac{1}{x^2} - 0\, dx$

$A = \int_1^5 x^{-2}\, dx$

$A = -\frac{1}{x} \Big|_1^5 = -\frac{1}{5} + 1 = \frac{4}{5}$

27) $f(y) = y^2$, $g(x) = y + 2$



$y^2 = y + 2$

$y^2 - y - 2 = 0$

$(y+1)(y-2) = 0$

$y = -1, y = 2$

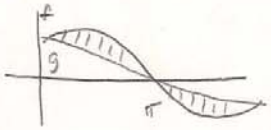
$A = \int_{-1}^{2} y + 2 - y^2\, dy$

$A = \frac{1}{2}y^2 + 2y - \frac{1}{3}y^3 \Big|_{-1}^{2}$

$A = \left(\frac{1}{2}(2)^2 + 2(2) - \frac{1}{3}(2)^3\right) - \left(\frac{1}{2}(-1)^2 + 2(-1) - \frac{1}{3}(-1)^3\right)$

$A = \left(2 + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right) = 7\frac{1}{2} - 3 = 4\frac{1}{2}$

36) $f(x) = \sin 2x$, $g(x) = \cos x$, $\frac{\pi}{6} \le x \le \frac{5\pi}{6}$



$A = \int_{\pi/6}^{\pi/2} \sin 2x - \cos x\, dx + \int_{\pi/2}^{5\pi/6} \cos x - \sin 2x\, dx$

$A = -\frac{1}{2}\cos 2x - \sin x \Big|_{\pi/6}^{\pi/2} + \sin x + \frac{1}{2}\cos 2x \Big|_{\pi/2}^{5\pi/6}$

$A = \left(\frac{1}{2} - 1\right) - \left(-\frac{1}{4} - \frac{1}{2}\right) + \left(\frac{1}{2} + \frac{1}{4}\right) - \left(1 - \frac{1}{2}\right)$

$A = -\frac{1}{2} + \frac{1}{4} + \frac{1}{2} + \frac{1}{2} + \frac{1}{4} - \frac{1}{2}$

$A = \frac{1}{2}$