sec. 6.4 Arc Length

Arc Length - the length of the curve between to points a & b.

$$s = \int_a^b \sqrt{1+[f'(x)]^2}\,dx \quad \text{where } y = f(x)$$

$$s = \int_c^d \sqrt{1+[g'(y)]^2}\,dy \quad \text{where } x = g(y)$$

Surface of Revolution

$$S = 2\pi \int_a^b r(x)\sqrt{1+[f'(x)]^2}\,dx$$

where $r(x)$ is the distance (radius) between the function & the axis of revolution [$r = f(x)$ @ x-axis, $r = x$ @ y-axis]

$$S = 2\pi \int_c^d r(y)\sqrt{1+[g'(y)]^2}\,dy$$

where $r(y)$ is the distance between the function and the axis of revolution [$r = g(y)$ @ y-axis, $r = y$ @ x-axis]

P.439

6) $y = \frac{x^5}{10} + \frac{1}{6x^3} \quad [1,2]$

$y' = \frac{x^4}{2} - \frac{1}{2x^4}$

$1+(y')^2 = 1 + \left(\frac{x^4}{2} - \frac{1}{2x^4}\right)^2$

$= \cancel{1} + \frac{x^8}{4} - \cancel{\frac{1}{2}} + \frac{1}{2x^8}$

$1+(y')^2 = \frac{1}{2} + \frac{x^8}{4} + \frac{1}{4x^8}$

$1+(y')^2 = \frac{x^{16}+2x^8+1}{4x^8}$

$1+(y')^2 = \left(\frac{x^8+1}{2x^4}\right)^2$

$s = \int_1^2 \sqrt{\left(\frac{x^4}{2} + \frac{1}{2x^4}\right)^2}\,dy$

$s = \int_1^2 \frac{x^4}{2} + \frac{1}{2}x^{-4}\,dx$

$s = \frac{1}{10}x^5 - \frac{1}{6x^3}\Big|_1^2 = \underline{\underline{\frac{779}{240}}}$

10) $y = \frac{1}{x+1}, \quad [0,1]$

$y' = -\frac{1}{(x+1)^2}$

$(y')^2 = \frac{1}{(x+1)^4}$

$s = \int_0^1 \sqrt{1+\left(\frac{1}{x+1}\right)^4}\,dx$

24) $y = 31 - 10\left(e^{x/20} + e^{-x/20}\right)$

$y' = \frac{10}{20}\left(e^{-x/20} - e^{x/20}\right)$

$s = \int_{-20}^{20} \sqrt{1+\left[\frac{1}{2}\left(e^{-\frac{x}{20}} + e^{\frac{x}{20}}\right)\right]^2}\,dx$

32) $y = \frac{x}{2} \quad [0,6]$

$y' = \frac{1}{2}$

$S = 2\pi \int_0^6 \frac{x}{2}\sqrt{1+\left(\frac{1}{2}\right)^2}\,dx$