sec. 3.9

Linear Approximations

to approximate a linear function to a given function $f(x)$ at a given point

① $y = f(c) + f'(c)(x-c)$

Differentials

When ① is used as an approximation to the graph of $f(x)$, $x-c$ is called the change in x, Δx. When Δx is small, the change in y, Δy, can be approximated by $\Delta y = f(x+\Delta x) - f(x)$ ②.

Δx, traditionally noted by dx is the differential of x

③ $dy = f'(x)\,dx$ is the differential of y. In many applications, the differential of y can be used to approximate the change in y. $\Delta y \approx dy$ or $\Delta y \approx f'(x)\,dx$.

Error $\frac{df}{f}$ is the relative error of a situation

$\frac{df}{f}(100)$ is the percentage error.

Approximating function values

④ $f(x+\Delta x) \approx f(x) + dy = f(x) + f'(x)\,dx$

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8) $y = 1 - 2x^2 \quad x = 0, \quad \Delta x = dx = -0.1$

$\Delta y = f(0-.1) - f(0)$

$\Delta y = .98 - 1$

$\underline{\Delta y = -.02}$

$dy = -4x\,dx$

$dy = -4(0)(-.1)$

$\underline{dy = 0}$

11) $y = 3x^2 - 4$

$\underline{dy = 6x\,dx}$

14) $y = (x^2-4)^{1/2}$

$dy = \frac{1}{2}(x^2-4)^{-1/2}(2x)\,dx$

$\underline{dy = x(x^2-4)^{-1/2}\,dx}$

18) $y = x\sin x$

$\underline{dy = (x\cos x + \sin x)\,dx}$

22) $A = \frac{1}{2}bh$, $b = 36\text{cm}$, $h = 50\text{cm}$

$db = dh = \pm .25\text{ cm}$

$\Delta A \approx dA = \frac{1}{2}b\,dh + \frac{1}{2}h\,db$

$dA = \frac{1}{2}(36)(\pm.25) + \frac{1}{2}(50)(\pm.25)$

$dA = \pm 4.5 + \pm 6.75$

$\underline{dA = \pm 10.75\text{ cm}^2}$

30) $V = \frac{4}{3}\pi r^3 \quad r = 100\text{cm} \quad dr = .2\text{cm}$

$dV = 4\pi r^2\,dr$

$dV = 4\pi(100\text{cm})^2(.2\text{cm})$

$\underline{dV = 8000\pi\text{ cm}^3}$