Essenital Concepts for sketching graphs

$$F(x) = aF(x - h) + k$$

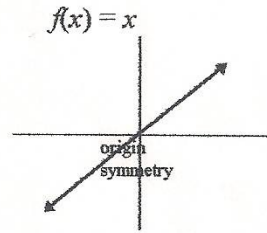| squeezes f(x) if $|a| > 1$ | causes lateral | causes vertical |
| stretches f(x) if $|a| < 1$ | shift of h units | shift of k units |
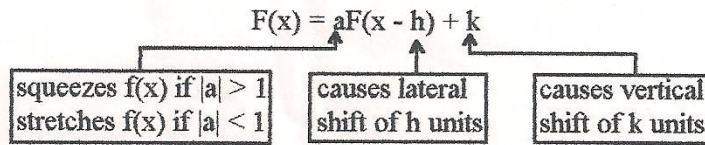
For all functions: x-intercept occurs where $f(x) = 0$ (i.e. $y = 0$)

y-intercept occurs at $f(0)$ (i.e. $x = 0$)

Linear Functions

$f(x) = ax + k$
or $y = mx + b$

the coefficient on the x term is always the slope
the constant term is always the y-intercept

$f(x) = x$

origin symmetry

Quadratic Functions
Parabolas
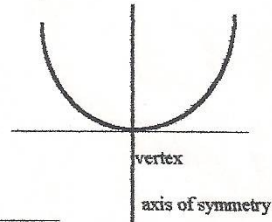
$f(x) = a(x - h)^2 + k$
or $f(x) = ax^2 + bx + c$

$a > 0$ parabola faces up
$a < 0$ parabola faces down
vertex located at (h, k)
k is extreme (min or max)
$x = h$ is axis of symmetry

$$h = -\frac{b}{2a} \qquad k = f(h)$$

$f(x) = x^2$

vertex
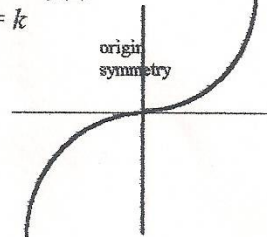axis of symmetry

x intercepts at $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Cubic Functions

$f(x) = a(x - h)^3 + k$

change in curvature at (h, k)
symmetry around $x = h$ & $y = k$

$f(x) = x^3$

origin symmetry

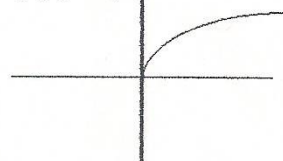### Square Root Functions

$f(x) = a\sqrt{x - h} + k$

Endpoint at (h, k)
Domain: $x \geq h$
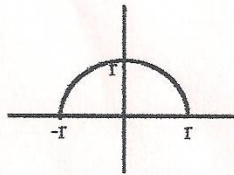Range: $y \geq k$ if $a > 0$
$y \leq k$ if $a < 0$

$f(x) = \sqrt{x}$

<u>Other typical radical functions</u>:

Half a Circle   Half a Hyperbola

$$f(x) = \sqrt{r^2 - x^2}$$ $$f(x) = \sqrt{x^2 - r^2}$$
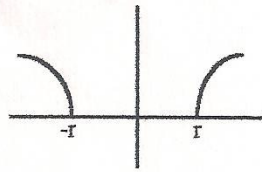
<u>Rational functions</u>

a) Linear $f(x) = \dfrac{a}{x - h} + k$ $f(x) = \dfrac{1}{x}$

Symmetry around $x = h$ & $y = k$
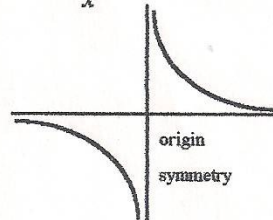Vertical Asymptote: $x = h$
Horizontal Asymptote: $y = k$
Domain: $x \neq h$
Range: $y \neq k$

origin
symmetry

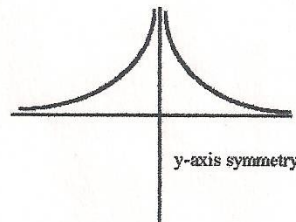b) Quadratic $f(x) = \dfrac{a}{(x - h)^2} + k$ $f(x) = \dfrac{1}{x^2}$

Asymptotes & Domain as above
Symmetry around $y = k$

Range: $\begin{cases} y > k, a > 0 \\ y < k, a < 0 \end{cases}$

y-axis symmetry

<u>Absolute value functions</u>
$$f(x) = a|x - h| + k$$ $$f(x) = |x|$$

Similar to parabolas
Vertex at (h, k)
In general, the graph of
$y = |f(x)|$ will be the same
as $y = f(x)$ with regions below
the x-axis rotated up