

3.9 Differential

defn: let $y = f(x)$ be differentiable on an open interval containing x .
 the differential of x (denoted by dx) is any nonzero real number.
 the differential of y (denoted by dy) is $dy = f'(x) dx$ ①

② $\Delta y \approx dy$ or $\Delta y \approx f'(x) dx$

③ $\Delta y = f(x + \Delta x) - f(x)$ [used to find propagated error, p. 233]

④ $f(x + \Delta x) \approx f(x) + dy = f(x) + f'(x) dx$ [used to approximate function values]

Error $\frac{dP}{P}$ is the relative error of situation; $\frac{dP}{P}(100)$ is percentage error.

P. 236

10) $y = 2x + 1$ $x = 2$ $\Delta x = dx = 0.01$
 use ① for dy use ③ for Δy

$dy = 2 dx$
 $dy = 2(0.01)$
 $= \underline{\underline{.02}}$

$\Delta y = f(2 + 0.01) - f(2)$
 $\Delta y = 5.02 - 5$
 $= \underline{\underline{.02}}$

4) $\sqrt[4]{624}$ $f(x) = \sqrt[4]{x}$ $x = 625$
 use ④ $\Delta x = -1$

$f(624) \approx \sqrt[4]{x} + \frac{1}{4} x^{-3/4} dx$
 $\approx \sqrt[4]{625} + \frac{1}{4\sqrt[4]{625^3}} (-1)$

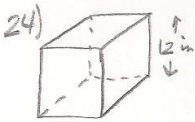
$\approx 5 - 0.002 \approx \underline{\underline{4.998}}$

by calculator: $\sqrt[4]{624} \approx \underline{\underline{4.9979}}$

16) $y = \sqrt{x} + \frac{1}{\sqrt{x}}$ use ①

$dy = \left(\frac{1}{2} x^{-1/2} - \frac{1}{2} x^{-3/2} \right) dx$

$dy = \left(\frac{1}{2\sqrt{x}} - \frac{1}{2\sqrt{x^3}} \right) dx$



use ①
 $x = 12 \text{ in}$ measured length
 $\Delta x = \pm .03 \text{ in}$ possible error

(a) $V = x^3$

$dV = 3x^2 dx$

$dV = 3(12 \text{ in})^2 (\pm .03 \text{ in})$

$= \pm 12.96 \text{ in}^3$

(b) $SA = 6x^2$

$dSA = 12x dx$

$= 12(12 \text{ in})(\pm .03 \text{ in})$

$= \pm 4.32 \text{ in}^2$