sec. 4.7 Indeterminate forms & L'Hopital's Rule

If $\lim_{x\to c} \frac{f(x)}{g(x)}$ produce the indeterminate form $\frac{0}{0}$, then $\lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f'(x)}{g'(x)}$.
provided the limit on the right exists (or is infinite).
Other indeterminate forms that allow you to apply L'Hopital's Rule:
$\frac{\infty}{\infty}$, $\frac{-\infty}{\infty}$, $\frac{\infty}{-\infty}$, or $\frac{-\infty}{-\infty}$.

Alternate form of L'Hopital: $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \lim_{x\to\infty} \frac{f'(x)}{g'(x)}$ provided
$\frac{f(x)}{g(x)} = \frac{0}{0}$ or $\frac{\infty}{\infty}$ and the limit on the right exists.

You may encounter forms such as $0\cdot\infty$, 1^{∞}, ∞^0, 0^0, or $\infty-\infty$. Your approach here is to algebraically manipulate the problem so it converts to $\frac{0}{0}$ or $\frac{\infty}{\infty}$.

4) $\lim_{x\to 0} \frac{\sin 4x}{2x}$ using $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1$

$2\lim_{x\to 0} \frac{\sin 4x}{4x} = 2(1) = \underline{\underline{2}}$ (Cal I)

8) $\lim_{x\to -1} \frac{x^2-x-2}{x+1}$

$\lim_{x\to -1} \frac{\cancel{(x+1)}(x-2)}{\cancel{x+1}} = \underline{\underline{-3}}$

14) $\lim_{x\to 1} \frac{\ln x}{x^2-1} = \frac{0}{0}$ use L'Hopital

$\lim_{x\to 1} \frac{\frac{1}{x}}{2x} = \underline{\underline{\frac{1}{2}}}$

20) $\lim_{x\to\infty} \frac{x-1}{x^2+2x+3} = \frac{\infty}{\infty}$ use L'Hopital

$\lim_{x\to\infty} \frac{1}{2x+2} = \frac{1}{\infty} = \underline{\underline{0}}$

28) $\lim_{x\to 0^+} x^2 \cot x = 0\cdot\infty$

$\lim_{x\to 0^+} \dfrac{x^2}{\tan x} = \dfrac{0}{0}$

$\lim_{x\to 0^+} \dfrac{2x}{\sec^2 x} = \dfrac{0}{1} = \underline{\underline{0}}$

32) $\lim_{x\to 0^+} (e^x + x)^{\frac{1}{x}} = 1^\infty$

let $y = \lim_{x\to 0^+} (e^x + x)^{\frac{1}{x}}$

then

$\ln y = \lim_{x\to 0^+} \ln(e^x + x)^{\frac{1}{x}}$

$\ln y = \lim_{x\to 0^+} \frac{1}{x} \ln(e^x + x) = \frac{0}{0}$

$\ln y = \lim_{x\to 0^+} \dfrac{\frac{1}{e^x + x}\cdot e^x + 1}{1} = \dfrac{\frac{2}{1}}{1}$

$e^{\ln y} = e^2$

$y = \underline{\underline{e^2}}$

40) $\lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{x^2}\right) = \infty - \infty$

$\lim_{x\to 0^+} \left(\dfrac{x-1}{x^2}\right) = \dfrac{-1}{0} = \underline{\underline{-\infty}}$

44) $\lim_{x\to\infty} \dfrac{x^3}{e^{2x}} = \dfrac{\infty}{\infty}$

$\lim_{x\to\infty} \dfrac{3x^2}{2e^{2x}} = \dfrac{\infty}{\infty}$

$\lim_{x\to\infty} \dfrac{6x}{4e^{2x}} = \dfrac{\infty}{\infty}$

$\lim_{x\to\infty} \dfrac{6}{8e^{2x}} = \dfrac{6}{\infty} = \underline{\underline{0}}$